PPLATO PROMOTING PHYSICS LEARNING & TEACHING OPPORTUNITIES

Basic Mathematics

Fractions

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The aim of this document is to provide a short, self assessment programme for students who wish to acquire a basic competence in the use of fractions.

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1. Fractions (Introduction)

The first thing to note is that all fractions can be represented in many different ways. Thus 3/12, 4/16, 5/20 represent the same fraction which, in its *lowest terms*, is 1/4.

Example 1 To show that each of the above fractions is equal to 1/4 we proceed as follows:

$$\frac{3}{12} = \frac{1\times 3}{4\times 3} = \frac{1\times \cancel{3}}{4\times \cancel{3}} = \frac{1}{4},$$

$$\frac{4}{16} = \frac{1\times 4}{4\times 4} = \frac{1\times \cancel{4}}{4\times \cancel{4}} = \frac{1}{4},$$

$$\frac{5}{20} = \frac{1\times 5}{4\times 5} = \frac{1\times \cancel{5}}{4\times \cancel{5}} = \frac{1}{4}.$$

The fractions in the left hand column each have a factor common to the numerator (top) and denominator (bottom) of that fraction, which is cancelled to give the fraction in its *lowest terms*.

In many problems it is necessary to carry out the reverse procedure, i.e. multiplying the numerator and denominator of a fraction by a common factor to obtain an *equivalent* fraction.

Example 2 Arrange each of the following fractions in the order of size.

$$(a)\ 3/4, \qquad (b)\ 2/3, \qquad (c)\ 5/6.$$

Solution To determine their relative order, each fraction must be written with the same denominator. The smallest such number is 12.

$$(a) \quad \frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12},$$

$$(b) \quad \frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12},$$

$$(c) \quad \frac{5}{6} = \frac{5 \times 2}{6 \times 2} = \frac{10}{12}.$$

The order of size, starting with the smallest, is 2/3, 3/4, 5/6.

EXERCISE 1. In each case below, arrange the fractions in increasing order of size. (Click on green letters for solution.)

(a) 7/12, 1/2, 2/3.

(b) 1/3, 5/8, 3/4.

(c) 5/6, 8/9, 11/12.

(d) 2/3, 3/5, 7/10.

Now try this short quiz. (Work out your solution and click on the green letter to see if you are correct.)

Quiz Of the following sets of fractions, exactly one is in ascending order of size. Which one?

(a) 1/3, 5/8, 3/5.

(b) 3/8, 2/3, 3/4.

(c) 2/3, 3/4, 1/2.

(d) 3/5, 5/7, 1/2.

2. Addition of Fractions

In this section we look at the addition (and subtraction) of fractions. If fractions are to be added then they must have the same denominators.

Example 3 Write the following sums of fractions as single fractions.

$$\text{(a)} \quad \frac{3}{8}+\frac{2}{3}-\frac{3}{4}, \qquad \text{(b)} \quad \frac{2}{3}+\frac{1}{2}-\frac{3}{4}.$$

Solution

(a) Taking all the fractions with denominator 24,

$$\begin{aligned}\frac{3}{8}+\frac{2}{3}-\frac{3}{4} &= \frac{3\times 3}{8\times 3}+\frac{2\times 8}{3\times 8}-\frac{3\times 6}{4\times 6}\\ &= \frac{9}{24}+\frac{16}{24}-\frac{18}{24}\\ &= \frac{9+16-18}{24}=\frac{7}{24}.\end{aligned}$$

(b) This time, taking all the fractions with denominator 12,

$$\begin{aligned}\frac{2}{3}+\frac{1}{2}-\frac{3}{4} &= \frac{2\times 4}{3\times 4}+\frac{1\times 6}{2\times 6}-\frac{3\times 3}{4\times 3}\\ &= \frac{8}{12}+\frac{6}{12}-\frac{9}{12}\\ &= \frac{8+6-9}{12}=\frac{5}{12}\,.\end{aligned}$$

The exercise below is designed to give you some practice at addition and subtraction of fractions.

EXERCISE 2. Evaluate the following, putting the final answer in its lowest terms. (Click on green letters for solution.)

(a) $\frac{1}{3}+\frac{1}{8}$, (b) $\frac{5}{6}+\frac{2}{4}$, (c) $\frac{3}{4}+\frac{4}{5}$,

(d) $\frac{4}{3}-\frac{1}{4}$, (e) $\frac{5}{6}-\frac{3}{2}$, (f) $\frac{5}{6}-\frac{3}{10}$.

To finish this section there follows two simple quizzes.

Quiz Which of the following fractions is the result of evaluating the sum

$$\frac{3}{4} - \frac{2}{3} + \frac{1}{6}\,?$$

(a) 1/4, (b) 1/3, (c) 1/5, (d) 3/8.

Quiz From the fraction given below, choose the one which is mid-way between 2/3 and 4/5.

(a) 3/4, (b) 3/5, (c) 10/15 (d) 11/15.

3. Multiplication and Division of Fractions

Multiplication of fractions is straightforward, as illustrated by the following.

Example 4 Evaluate the following products.

$$\text{(a)}\ \frac{2}{5}\times\frac{3}{4}, \qquad \text{(b)}\ \frac{3}{4}\times\frac{1}{3}.$$

Solution

It is simply a matter of multiplying the **numerators** and **denominators**.

$$\textbf{(a)}\ \frac{2}{5}\times\frac{3}{4}=\frac{2\times 3}{5\times 4}=\frac{6}{20}=\frac{3}{10}, \qquad \textbf{(b)}\ \frac{3}{4}\times\frac{1}{3}=\frac{3\times 1}{4\times 3}=\frac{1}{4}.$$

In each case the **common factors** have been cancelled to leave the fraction in its **lowest form**.

Division of fractions is straightforward once we note that **division** by a fraction of the form a/b is equivalent to **multiplication** by the fraction b/a. This can be seen in the following example.

Example 5 Write the following as fractions in the usual way.

$$\textbf{(a)}\ 1 \div \left(\tfrac{3}{4}\right), \qquad \textbf{(b)}\ \left(\tfrac{1}{8}\right) \div \left(\tfrac{3}{4}\right).$$

Solution

(a) First note that

$$\left(\frac{3}{4}\right) \times \left(\frac{4}{3}\right) = \frac{3 \times 4}{4 \times 3} = \frac{12}{12} = 1\,.$$

Divide both sides of this equation by $(3/4)$.

$$\begin{aligned} \left(\frac{3}{4}\right) \times \left(\frac{4}{3}\right) &= 1 \\ \therefore \left(\frac{4}{3}\right) &= 1/\left(\frac{3}{4}\right) = 1 \div \left(\frac{3}{4}\right). \end{aligned}$$

(b) The previous result may be used.

$$\left(\frac{1}{8}\right) \div \left(\frac{3}{4}\right) = \frac{\left(\frac{1}{8}\right)}{\left(\frac{3}{4}\right)} = \frac{\left(\frac{1}{8}\right) \times 1}{1 \times \left(\frac{3}{4}\right)} = \frac{\left(\frac{1}{8}\right)}{1} \times \frac{1}{\left(\frac{3}{4}\right)}$$

$$= \left(\frac{1}{8}\right) \times \left(\frac{4}{3}\right) \text{ using } \textbf{(a)}$$

$$= \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}\,.$$

where a common factor 4 has been cancelled.

EXERCISE 3. Evaluate each of the following in their lowest terms.

(a) $\left(\frac{1}{3} + \frac{1}{8}\right) \times \frac{2}{11}$,

(b) $\left(\frac{5}{6} + \frac{1}{2}\right) \div \frac{1}{3}$,

(c) $\frac{\left(\frac{3}{4} + \frac{4}{5}\right)}{\left(\frac{1}{4} - \frac{1}{5}\right)}$,

(d) $\frac{\left(\frac{5}{6} - \frac{3}{2}\right)}{\left(\frac{4}{3} - \frac{1}{4}\right)}$.

4. Quiz on Fractions

Begin Quiz Evaluate each of the following and choose the answer from those given below.

1. $\left(\frac{3}{4} - \frac{2}{3}\right) \div \left(\frac{3}{4}\right)$

(a) 2/9, (b) 1/6, 5/12, (c) (d) 1/9.

2. $\left(\frac{3}{5} + \frac{1}{3}\right) \times \left(\frac{5}{7}\right)$

(a) 1/7, (b) 2/3, (c) 2/15, (d) 4/21.

3. $\left(\frac{3}{8} - \frac{1}{5}\right) \div \left(\frac{7}{10} - \frac{2}{3}\right)$

(a) 4/15, (b) 13/25, (c) 13/20, (d) 21/4.

End Quiz Score: Correct

Solutions to Exercises

Exercise 1(a) The *least common denominator* of the fractions is 12, so

$$\frac{1}{2} = \frac{1 \times 6}{2 \times 6} = \frac{6}{12},$$

$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}.$$

Since 7/12 already has 12 as denominator, the required order of the fractions is

$$\frac{1}{2}, \frac{7}{12}, \frac{2}{3}.$$

Click on green square to return

Exercise 1(b) In this case, the *least common denominator* of the fractions is 24.

$$\frac{1}{3} = \frac{1 \times 8}{3 \times 8} = \frac{8}{24},$$

$$\frac{5}{8} = \frac{5 \times 3}{8 \times 3} = \frac{15}{24},$$

$$\frac{3}{4} = \frac{3 \times 6}{4 \times 6} = \frac{18}{24}.$$

The fractions are thus increasing in the order in which they appear.

Click on green square to return

Exercise 1(c) In this case the *least common denominator* of the fractions is 36.

$$\frac{5}{6} = \frac{5 \times 6}{6 \times 6} = \frac{30}{36},$$

$$\frac{8}{9} = \frac{8 \times 4}{9 \times 4} = \frac{32}{36},$$

$$\frac{11}{12} = \frac{11 \times 3}{12 \times 3} = \frac{33}{36},$$

The fractions are thus increasing in the order in which they appear.

Click on green square to return □

Exercise 1(d) In this case the *least common denominator* of the fractions is 30.

$$\frac{2}{3} = \frac{2 \times 10}{3 \times 10} = \frac{20}{30},$$

$$\frac{3}{5} = \frac{3 \times 6}{5 \times 6} = \frac{18}{30},$$

$$\frac{7}{10} = \frac{7 \times 3}{10 \times 3} = \frac{21}{30}.$$

The fractions, in increasing order of size, are 3/5, 2/3, 7/10.

Click on green square to return

Exercise 2(a) The *lowest common denominator* is 24, so

$$\begin{aligned}\frac{1}{3}+\frac{1}{8} &= \frac{1\times 8}{3\times 8}+\frac{1\times 3}{8\times 3}\\ &= \frac{8}{24}+\frac{3}{24}\\ &= \frac{8+3}{24}=\frac{11}{24}\,.\end{aligned}$$

Click on green square to return

Exercise 2(b) Before proceeding, note that the second fraction is not in its lowest terms. Since $2/4=(1\times 2)/(2\times 2)=1/2$,

$$\begin{aligned}
\frac{5}{6}+\frac{2}{4} &= \frac{5}{6}+\frac{1}{2} \\
&= \frac{5}{6}+\frac{1\times 3}{2\times 3} \\
&= \frac{5}{6}+\frac{3}{6} \\
&= \frac{5+3}{6}=\frac{8}{6} \\
&= \frac{4\times 2}{3\times 2}=\frac{4}{3}\,.
\end{aligned}$$

This fraction is called an *improper fraction* since the numerator is larger than the denominator. It is perfectly acceptable as a fraction but it may also written as $1\frac{1}{3}$.

Click on green square to return □

Exercise 2(c)

The *least common denominator* of the two fractions is 20 so

$$\begin{aligned}\frac{3}{4}+\frac{4}{5} &= \frac{3\times 5}{4\times 5}+\frac{4\times 4}{5\times 4}\\ &= \frac{15}{20}+\frac{16}{20}\\ &= \frac{15+16}{20}=\frac{31}{20}\,.\end{aligned}$$

This is another *improper fraction* which may be written as $1\frac{11}{20}$.

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Exercise 2(d)

The *least common denominator* of the two fractions is 12 so

$$\begin{aligned}\frac{4}{3}-\frac{1}{4} &= \frac{4\times 4}{3\times 4}-\frac{1\times 3}{4\times 3}\\ &= \frac{16}{12}-\frac{3}{12}\\ &= \frac{16-3}{12}=\frac{13}{12}.\end{aligned}$$

This is another *improper fraction* which is equal to $1\frac{1}{12}$..

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Exercise 2(e)

The *least common denominator* of the two fractions is 6 so

$$\begin{aligned}
\frac{5}{6} - \frac{3}{2} &= \frac{5}{6} - \frac{3 \times 3}{2 \times 3} \\
&= \frac{5}{6} - \frac{9}{6} \\
&= \frac{5-9}{6} = -\frac{4}{6} \\
&= -\frac{2 \times 2}{3 \times 2} = -\frac{2}{3}\,.
\end{aligned}$$

where the common factor 2 has been cancelled to obtain the final answer.

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Exercise 2(f)

The *least common denominator* of the two fractions is 30 so

$$\begin{aligned}
\frac{5}{6} - \frac{3}{10} &= \frac{5 \times 5}{6 \times 5} - \frac{3 \times 3}{10 \times 3} \\
&= \frac{25}{30} - \frac{9}{30} \\
&= \frac{25 - 9}{30} = \frac{16}{30} \\
&= \frac{8 \times 2}{15 \times 2} = \frac{8}{15},
\end{aligned}$$

where the final answer is obtained after cancellation of the common factor 2.

Click on green square to return

Exercise 3(a) Using exercise 2 part **(a)** we have

$$\begin{aligned}\left(\frac{1}{3}+\frac{1}{8}\right)\times\frac{2}{11} &= \frac{11}{24}\times\frac{2}{11}\\ &= \frac{2}{24}=\frac{1}{12}\,,\end{aligned}$$

where the common factors of 11 and then 2 have been cancelled.

Click on green square to return □

Exercise 3(b)

Using the result of exercise 2 part **(b)** we have

$$\begin{aligned}\left(\frac{5}{6}+\frac{1}{2}\right)\div\frac{1}{3} &= \frac{4}{3}\div\frac{1}{3}\\ &= \frac{4}{3}\times\frac{3}{1}\\ &= \frac{4}{1}=4\,,\end{aligned}$$

after recalling that division by 1/3 is the same as multiplication by 3/1=3, and cancelling the common factor 3.

Click on green square to return

Exercise 3(c) From exercise 2 part **(c)**, the numerator is 31/20. The denominator is evaluated below.

$$\begin{aligned}\frac{1}{4}-\frac{1}{5} &= \frac{1\times 5}{4\times 5}-\frac{1\times 4}{5\times 4}\\ &= \frac{5}{20}-\frac{4}{20}=\frac{1}{20}\,.\end{aligned}$$

Thus

$$\begin{aligned}\frac{\left(\frac{3}{4}+\frac{4}{5}\right)}{\left(\frac{1}{4}-\frac{1}{5}\right)} &= \frac{\left(\frac{31}{20}\right)}{\left(\frac{1}{20}\right)}\\ &= \left(\frac{31}{20}\right)\times\left(\frac{20}{1}\right)\\ &= 31\,,\end{aligned}$$

after cancellation of the common factor 20.

Click on green square to return

Exercise 3(d) In this case, from exercise 2 part **(c)**, the denominator is 13/12. Evaluating the numerator,

$$\begin{aligned}\frac{5}{6}-\frac{3}{2} &= \frac{5}{6}-\frac{9}{6}\\ &= -\frac{4}{6}=-\frac{2}{3}\,.\end{aligned}$$

after cancelling the common factor 2. Thus

$$\begin{aligned}\frac{\left(\frac{5}{6}-\frac{3}{2}\right)}{\left(\frac{4}{3}-\frac{1}{4}\right)} &= -\frac{\left(\frac{2}{3}\right)}{\left(\frac{13}{12}\right)}=-\left(\frac{2}{3}\right)\times\left(\frac{12}{13}\right)\\ &= -\left(\frac{2}{3}\right)\times\left(\frac{4\times 3}{13}\right)\\ &= -\left(\frac{2}{1}\right)\times\left(\frac{4\times 1}{13}\right)=-\frac{8}{13}\,,\end{aligned}$$

after cancellation of the common factor 3.

Click on green square to return □

Solutions to Quizzes

Solution to Quiz: Putting each of them in the form of a fraction with denominator 24,

$$\frac{3}{8} = \frac{3\times 3}{8\times 3} = \frac{9}{24},$$

$$\frac{2}{3} = \frac{2\times 8}{3\times 8} = \frac{16}{24},$$

$$\frac{3}{4} = \frac{3\times 6}{4\times 6} = \frac{18}{24}.$$

These are obviously in ascending order. Repeating this with the others will show that this is the only set in ascending order.

End Quiz

Solution to Quiz:
The *least common denominator* of the three fractions is 12, so

$$\begin{aligned}
\frac{3}{4}-\frac{2}{3}+\frac{1}{6} &= \frac{3\times 3}{4\times 3}-\frac{2\times 4}{3\times 4}+\frac{1\times 2}{6\times 2}\\
&= \frac{9}{12}-\frac{8}{12}+\frac{2}{12}\\
&= \frac{9-8+2}{12}=\frac{11-8}{12}\\
&= \frac{3}{12}=\frac{1\times 3}{4\times 3}=\frac{1}{4}.
\end{aligned}$$

End Quiz

Solution to Quiz: The *least common denominator* of the two fractions is 15. Writing both fractions with this denominator gives

$$\frac{2}{3} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15},$$
$$\frac{4}{5} = \frac{4 \times 3}{5 \times 3} = \frac{12}{15}.$$

The fraction mid-way between 10/15 and 12/15 is 11/15.

End Quiz